

Quiz #1 (Tuesday)

- laws for exponentials/logarithms
- sketch trig function, determine value of inverse

Review. compound interest, APR vs APY, $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$

Example 12. (real life) My car loan contract quotes an APR of 6.79% (and compounds monthly). What is the APY?

Solution. Since $\left(1 + \frac{0.0679}{12}\right)^{12} \approx 1.07005$, the APY is about 7.01%.

[Obviously, the lender prefers to quote the lower but less meaningful APR.]

Limits and Continuity

$$\lim_{x \rightarrow c} f(x) = L$$

means that $f(x)$ approaches the value L as x approaches the value c .

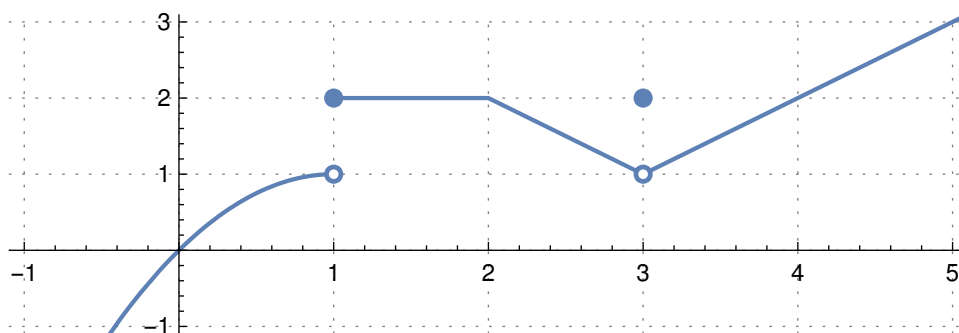
More precisely, $f(x)$ is as close to L as we want for all x (different from c but) sufficiently close to c .

We will soon make it very precise what we mean by “approach”! For now, let us build some first intuition.

Similarly, we have the one-sided limits

- $\lim_{x \rightarrow c^-} f(x) = L$ for the case when x approaches c from the left, and
- $\lim_{x \rightarrow c^+} f(x) = L$ for the case when x approaches c from the right.

Example 13. For $f(x)$ as sketched below, discuss the limits $\lim_{x \rightarrow c} f(x)$ for $c = 1, 2, 3, 4$ as well as the corresponding one-sided limits.



Solution.

- $(c=1)$ $\lim_{x \rightarrow 1^-} f(x) = 1$ while $\lim_{x \rightarrow 1^+} f(x) = 2$, so that $\lim_{x \rightarrow 1} f(x)$ does not exist.
[Note that $f(1) = 2$, but that this is irrelevant for the limits.]
- $(c=2)$ $\lim_{x \rightarrow 2} f(x) = 2$ (which automatically implies that $\lim_{x \rightarrow 2^-} f(x) = 2$ and $\lim_{x \rightarrow 2^+} f(x) = 2$ as well)
- $(c=3)$ $\lim_{x \rightarrow 3} f(x) = 1$ (which automatically implies that $\lim_{x \rightarrow 3^-} f(x) = 1$ and $\lim_{x \rightarrow 3^+} f(x) = 1$ as well)
[Note that $f(3) = 2$, but that this is irrelevant for the limits.]
- $(c=4)$ $\lim_{x \rightarrow 4} f(x) = 2$ (which automatically implies that $\lim_{x \rightarrow 4^-} f(x) = 2$ and $\lim_{x \rightarrow 4^+} f(x) = 2$ as well)

Example 14. Sketch a function with $\lim_{x \rightarrow 2^-} f(x) = 2$, $\lim_{x \rightarrow 2^+} f(x) = 1$ and $f(2) = 3$.

Solution. The possibilities are endless. Here's one such $f(x)$:

