Example 116. Determine $\lim_{x\to 0} \frac{8^x - 1}{2^x - 1}$.

Solution. $\lim_{x\to 0} \frac{8^x - 1}{2^x - 1} \overset{\text{LH}}{\underset{“\frac{0}{0}”}{=}} \lim_{x\to 0} \frac{\ln(8)\, 8^x}{\ln(2)\, 2^x} = \frac{\ln(8)}{\ln(2)} = \log_2(8) = 3$.

Example 117. Determine $\lim_{x\to\infty} (\ln(x))^{1/x}$.

Solution. $\lim_{x\to\infty} \ln((\ln(x))^{1/x}) = \lim_{x\to\infty} \frac{\ln(\ln(x))}{x} \overset{\text{LH}}{\underset{“\frac{\infty}{\infty}”}{=}} \lim_{x\to\infty} \frac{\frac{1}{\ln(x)} \cdot \frac{1}{x}}{1} = 0$.
Hence, $\lim_{x\to\infty} (\ln(x))^{1/x} = e^0 = 1$.

[... April Fools' Day ...]

Estimating areas

For much more detail (using a very slightly different example), as well as nice illustrations, you are strongly encouraged to read through the beginning of Section 5.1 in our book.

Example 118. Sketch the area A between the x-axis and $f(x) = 4 - x^2$ for x in $[0, 2]$. Then estimate the area by dividing $[0, 2]$ into four subintervals, and using each subinterval as the base of a rectangle whose height is

(a) the maximum of f on the subinterval (“upper sum”),

(b) the minimum of f on the subinterval (“lower sum”),

(c) the values of f at the center of the subinterval (“midpoint rule”).

Solution. The four subintervals are $\left[0, \frac{1}{2}\right]$, $\left[\frac{1}{2}, 1\right]$, $\left[1, \frac{3}{2}\right]$, $\left[\frac{3}{2}, 2\right]$.

(a) $A < \frac{1}{2}\left[f(0) + f\left(\frac{1}{2}\right) + f(1) + f\left(\frac{3}{2}\right)\right] = \frac{1}{2}\left[4 + \frac{15}{4} + 3 + \frac{7}{4}\right] = \frac{16 + 15 + 12 + 7}{8} = \frac{25}{4} = 6.25$

(b) $A > \frac{1}{2}\left[f\left(\frac{1}{2}\right) + f(1) + f\left(\frac{3}{2}\right) + f(2)\right] = \frac{1}{2}\left[\frac{15}{4} + 3 + \frac{7}{4} + 0\right] = \frac{17}{4} = 4.25$

(c) $A \approx \frac{1}{2}\left[f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right) + f\left(\frac{5}{4}\right) + f\left(\frac{7}{4}\right)\right] = \frac{43}{8} = 5.375$

Comment. The exact area is $A = \frac{16}{3} \approx 5.333$.

Armin Straub
straub@southalabama.edu