Practice for Midterm #2

MATH 125 — Calculus 1
Tuesday, March 12

Please print your name:

Besides the allowed calculator, no notes or tools of any kind will be permitted.

- Have another look at the homework, especially those problems that you struggled with.
- Retake Quizzes 4, 5, and 6! (Versions with and without solutions are posted to our course website.)
- Go through the lecture sketches (posted to our course website) and do the problems we did in class (ignore the solutions until you have solved the problem yourself).

Problem 1. Compute the following derivatives.

(a) $\frac{\mathrm{d}}{\mathrm{d}x}[2x^4 - 3\sqrt{x} + 7x - 4^2]$

(b) $\frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{x^7\sin(4x+5)}$

(c) $\frac{\mathrm{d}}{\mathrm{d}x}e^{x^2-3}$

(d) $\frac{\mathrm{d}}{\mathrm{d}x}[x^3\ln(x^5 + \cos(2x))]$

(e) $\frac{\mathrm{d}}{\mathrm{d}x}\sin^{-1}\left(x^2 + \frac{1}{x}\right)$

(f) $\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{1 + (x^2-1)\tan^{-1}(3x+2)}$

Problem 2. Compute the derivatives of the following functions.

(a) $(\cos(x))^x$

(b) $x^{\cos(x)}$

Problem 3. State the limit definition of $f'(3)$.

Problem 4. The Lambert W function is defined as the inverse function of $f(x) = xe^x$. Determine $W'(x)$.

Problem 5. Let $f(x) = \frac{1}{x^2 + g(-x)}$ and suppose that $g(-1) = 2$ and $g'(-1) = 3$. Find $f'(1)$.

Problem 6. Consider the curve $y^2\cos(x) + 2xy^3 = 4$.

(a) Using implicit differentiation, determine $\frac{\mathrm{d}y}{\mathrm{d}x}$.

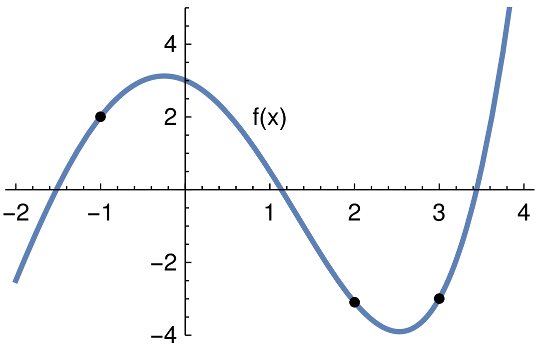
(b) Determine the line tangent to the curve at the point $(0, 2)$.

(c) Determine the line normal to the curve at the point $(0, 2)$.

Problem 7. Consider the curve $xy + y^2 = 1$. Determine $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

Armin Straub
straub@southalabama.edu

Problem 8. Use the graph below to fill in each entry of the grid with positive, negative or zero.



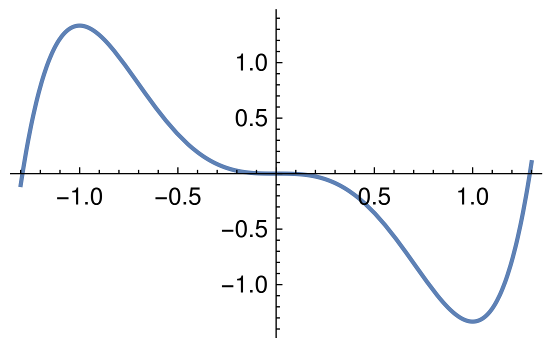
	$f(x)$	$f'(x)$	$f''(x)$
$x=-1$			
$x=2$			
$x=3$			

Problem 9. The plot to the right shows the function $f(x)$.

(a) What are the critical points of $f(x)$?

(b) Mark (roughly) the inflection points of $f(x)$ in the plot.

(c) $g(x)$ is a function such that $g'(x)=f(x)$. Does $g(x)$ have a local extremum at $x=0$?



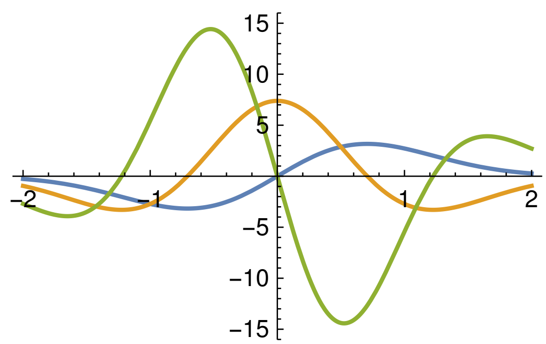
Problem 10. Roughly sketch a differentiable function $f(x)$ with the following property.

(a) $f'(0)=0$ but 0 is not a local extremum,

(b) $f'(0)>0$ and $f''(0)<0$.

Problem 11. The plot to the right shows a function $f(x)$ as well as $f'(x)$ and $f''(x)$.

Which is which? Label the graphs accordingly.



Problem 12. The first and second derivatives of the function $f(x)$ have the following values:

	$x<-2$	$x=-2$	$-2<x<-1$	$x=-1$	$-1<x<0$	$x=0$	$0<x<1$	$x=1$	$1<x<3$	$x=3$	$x>3$
$f'(x)$	$-$	0	$+$	$+$	$+$	0	$+$	$+$	$+$	0	$-$
$f''(x)$	$+$	$+$	$+$	0	$-$	0	$+$	0	$-$	0	$-$

Determine the location of all local minima, local maxima and inflection points.

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Problem 13. Determine all local extrema of the function $f(x) = x^4 - \frac{4}{3}x^3 - 4x^2 + 24x + 1$. You may use that the critical points are at $x = -1$, $x = 0$ and $x = 2$.

Problem 14. Consider the function $s(t) = 2t^3 - 9t^2 + 12t$.

(a) What is the maximal value $s(t)$ attained on the interval $\left[0, \frac{1}{2}\right]$?

(b) Determine all local extrema of $s(t)$.

(c) On which intervals is $s(t)$ increasing?

(d) On which intervals is $s(t)$ concave up?

(e) Determine all inflection points of $s(t)$.

Problem 15. Consider $f(x) = (x^2 - 2)e^{2x}$.

(a) Determine all local and absolute extrema of $f(x)$ on the interval $[-3, 3]$.

(b) Determine the inflection points of $f(x)$.

Problem 16. The volume of a cube is increasing at a rate of 10 cm^3/min. How fast is the surface area increasing when the length of an edge is 40 cm?

Problem 17. Oil is leaking from a tanker and spreads in a circle whose area increases at a rate of 5 km^2/h. How fast is the radius of the spill increasing after 4 h?

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