

Practice for Midterm #3

MATH 125 — Calculus 1

Tuesday, April 16

Please print your name:

Besides the allowed calculator, no notes or tools of any kind will be permitted.

- Have another look at the homework sets #10, #11, #12, #13, especially those problems that you struggled with.
- Retake Quizzes 7 and 8! (Versions with and without solutions are posted to our course website.)
- Go through the lecture sketches (posted to our course website) and do the problems we did in class (ignore the solutions until you have solved the problem yourself).

Problem 1. Compute the following limits.

(a) $\lim_{x \rightarrow \infty} x e^{-2x}$

(d) $\lim_{x \rightarrow 0} \frac{\cos(2x) - 1}{\sin(3x^2)}$

(b) $\lim_{x \rightarrow \infty} \frac{\sqrt{x}}{\ln(x^4 + 1)}$

(e) $\lim_{x \rightarrow \infty} \left(1 - \frac{3}{x^2}\right)^x$

(c) $\lim_{x \rightarrow 0} \frac{\cos(2x) - 1}{\sin(3x)}$

(f) $\lim_{x \rightarrow 1} \left(\frac{1}{\ln(x)} - \frac{1}{x-1}\right)$

Problem 2. Evaluate the following indefinite integrals.

(a) $\int (x^4 + 3x^2 - 7x + 1) dx$

(c) $\int (e^{2x} - 7e^{-3x}) dx$

(b) $\int \left(\sqrt{x} - \frac{1}{x^4}\right) dx$

(d) $\int \frac{\sin(4x) + 2\cos(3x)}{5} dx$

Problem 3. Evaluate the following definite integrals.

(a) $\int_2^3 (x^2 - 2) dx$

(d) $\int_1^2 \left(\frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}\right) dx$

(b) $\int_1^4 \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx$

(e) $\int_0^{\pi/2} \sin(2x) dx$

(c) $\int_0^3 e^{4x} dx$

Problem 4. Suppose you have 160 m of fencing and want to fence off a rectangular field that borders a straight river (no fence is needed alongside the river). What is the maximum area you can fence off?

Problem 5. You are designing a rectangular poster to contain 50 in^2 of printing with a 4 in margin at the top and bottom, and a 2 in margin at each side. What overall dimensions will minimize the amount of paper used?

Problem 6. Evaluate the following sums.

$$(a) \sum_{k=3}^6 2^{-k}(1 - (-1)^k)$$

$$(b) \sum_{k=1}^4 \frac{(-1)^k}{\sqrt{k}}$$

Problem 7. Let A be the area between the x -axis and $f(x)$ for x in $[2, 6]$.

(a) Write down a Riemann sum for A using 5 intervals (of equal size) and

(1) left endpoints,

(2) right endpoints,

(3) midpoints.

(b) Using sigma notation, write down a Riemann sum for A using n intervals (of equal size) and

(1) left endpoints,

(2) right endpoints,

(3) midpoints.

Problem 8. Let A be the area between the x -axis and $f(x) = \sqrt{x}$ for x in $[1, 5]$.

(a) Estimate the area A using a Riemann sum with 3 intervals and left endpoints.

(b) Estimate the area A using a Riemann sum with 3 intervals and midpoints.

(c) Compute the (exact) area A .

Problem 9.

(a) Estimate the average value of $f(x) = \frac{1}{x^2}$ on $[1, 3]$ using a Riemann sum with 3 intervals and midpoints.

(b) Compute the (exact) average value of $f(x) = \frac{1}{x^2}$ on $[1, 3]$.