Quiz #3

MATH 125 — Calculus 1
Tuesday, Jan 29

Please print your name:

Problem 1. (4 points) For what values of a is $f(x) = \begin{cases} 3x - a, & x < 2, \\ ax^2 + 1, & x \geqslant 2, \end{cases}$ continuous at every x? [Show work!]

Solution. Observe that $f(x)$ is always continuous at every point except, possibly, $x = 2$. (Why?!)

In order for $f(x)$ to be continuous at $x = 2$, we need $\lim_{x \to 2} f(x) = f(2)$.

- $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (3x - a) = 6 - a$
- $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (ax^2 + 1) = 4a + 1 = f(2)$

Hence, $\lim_{x \to 2} f(x) = f(2)$ if and only if $6 - a = 4a + 1$, which happens if and only if $a = 1$.

Thus, $f(x)$ is continuous if and only if $a = 1$. □

Problem 2. (1+3 points) Let $f(x)$ be a complicated continuous function taking the following values:

x	-3	-2	-1	0	1	2	3
$f(x)$	2	3	1	-1	-3	4	4

(a) What can we conclude about solutions to the equation $f(x) = 0$ for x in the interval $[2, 3]$? [select one]

- ☐ There is exactly one solution in the interval $[2, 3]$.
- ☐ There is at least one solution in the interval $[2, 3]$.
- ☐ There is no solution in the interval $[2, 3]$.
- ☐ There might or might not be a solution in the interval $[2, 3]$.

(b) Using the intermediate value theorem, what can we conclude about solutions to the equation $f(x) = 0$?

We can guarantee that there is a solution in the following intervals: [list intervals that are as small as possible]

Solution.

(a) There might or might not be a solution in the interval $[2, 3]$.

(b) We can guarantee that there is a solution in the intervals $[-1, 0]$ and $[1, 2]$.

[If we wanted, we could use open intervals and say that there is a solution in the intervals $(-1, 0)$ and $(1, 2)$.] □

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