(Bonus) Quiz #7

MATH 125 — Calculus 1
Friday, March 15

Please print your name:

Problem 1. (2 points) Compute the following derivatives. [No need to show work.]

(a) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{1}{\sqrt{x}} + e^3\right] =$ [ ]

(b) $\frac{\mathrm{d}}{\mathrm{d}x}\ln(\sin(3x)) =$ [ ]

Solution.

(a) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{1}{\sqrt{x}} + \frac{1}{\sqrt{\pi}}\right] = -\frac{1}{2}x^{-3/2}$

(b) $\frac{\mathrm{d}}{\mathrm{d}x}\ln(\sin(3x)) = \frac{3\cos(3x)}{\sin(3x)} = 3\cot(3x)$ (the last step is optional) □

Problem 2. (3+2 points) Consider the function $f(x) = (x+1)e^{3x}$. [Show your work!]

(a) $f(x)$ has local maxima at $x =$ [ ] and local minima at $x =$ [ ]. [or write "none"]

(b) $f(x)$ has inflection points at $x =$ [ ]. [or write "none"]

Solution.

(a) Because the derivatives of $f(x)$ are pleasant to compute, we will use the second-derivative test.
Since $f'(x) = e^{3x} + 3(x+1)e^{3x} = (3x+4)e^{3x}$, the only critical point is at $x = -\frac{4}{3}$.
$f''(x) = 3e^{3x} + 3(3x+4)e^{3x} = 3(3x+5)e^{3x}$
Since $f''\left(-\frac{4}{3}\right) = 3e^{-4} > 0$, $f(x)$ has a local min at $x = -\frac{4}{3}$.

(b) Solving $f''(x) = 0$, we find $x = -\frac{5}{3}$. To see that the concavity is indeed changing at $x = -\frac{5}{3}$, we can check $f''(-2) = -3e^{-6} < 0$ and $f''(0) = 15 > 0$. Hence, $f(x)$ has an inflection point at $x = -\frac{5}{3}$. □

Problem 3. (3 points) Oil is leaking from a tanker and spreads in a circle whose area increases at a constant rate of 7 km^2/h. How fast is the radius of the spill increasing after 4 h?

Solution. Let A be the area (in km^2) and r the radius (in km) of the circular spill. Then A and r are related by the equation $A = \pi r^2$. It follows that the rates of change, with respect to time t (in h), are related by

$$\frac{\mathrm{d}A}{\mathrm{d}t} = 2\pi r\frac{\mathrm{d}r}{\mathrm{d}t}.$$

We have $\frac{\mathrm{d}A}{\mathrm{d}t} = 7$. After $t = 4$, the area is $A = 4 \cdot 7$, so that the radius is $r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{28}{\pi}}$. It follows that

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{1}{2\pi r}\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{7}{2\pi\sqrt{\frac{28}{\pi}}} = \frac{1}{4}\sqrt{\frac{7}{\pi}} \approx 0.373 \text{ km/h}.$$

□

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