Midterm #3

Please print your name:

No notes, graphing calculators or other tools are permitted.   There are 34 points in total.   You need to show work to receive full credit.

Good luck!

Problem 1. (6 points) Compute the following series (or state that it diverges):

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{6^n - 2}{3^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{6 - 2^n}{3^n}$

Problem 2. (6 points) Determine the Taylor polynomial of order 3 for $f(x) = \sqrt{x}$ at $x = 1$.

Problem 3. (4 points) Determine the following limits or state that the limit does not exist.

(a) $\qquad \lim\limits_{n\to\infty} \dfrac{5^n + 3^n}{4^n - 1} =$

(b) $\qquad \lim\limits_{n\to\infty} \dfrac{7n^2 - 8n}{2n^2 + 3} =$

(c) $\qquad \lim\limits_{n\to\infty} \sqrt{\dfrac{3 + 2n^2}{1 + n + n^2}} =$

(d) $\qquad \lim\limits_{n\to\infty} \cos\left(\dfrac{n}{n^2 + 1}\right) =$

Problem 4. (8 points) Determine whether the following series converge or diverge.　Make sure to indicate a reason!

(a) $\qquad \sum\limits_{n=2}^{\infty} \dfrac{1 - \log(n)}{1 + \log(n)}$

☐ series converges　　　　☐ series diverges

Indicate a reason:

(b) $\qquad \sum\limits_{n=1}^{\infty} \dfrac{n + 1}{n^3 + 1}$

☐ series converges　　　　☐ series diverges

Indicate a reason:

(c) $\qquad \sum\limits_{n=2}^{\infty} \dfrac{7^n}{n^2 \, 4^n}$

☐ series converges　　　　☐ series diverges

Indicate a reason:

(d) $\qquad \sum\limits_{n=2}^{\infty} \dfrac{n + \sqrt{n} + 7}{3n^2 + 1}$

☐ series converges　　　　☐ series diverges

Indicate a reason:

Problem 5. (10 points) Consider the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(x+1)^n}{\sqrt{n}\, 3^n}$.

   (a) Determine the radius of convergence R.

   (b) What is the exact interval of convergence?

   (c) Let $\displaystyle f(x) = \sum_{n=1}^{\infty} \frac{(x+1)^n}{\sqrt{n}\, 3^n}$ for x such that $|x+1| < R$. Write down a power series for $f'(x)$.

Problem 6. (Bonus!) What is the value of $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$?   [We don't have the tools to evaluate this series, but you might remember from class.]

(extra scratch paper)

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