

# Quiz #6

Please print your name:

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**Problem 1. (4 points)** Determine the shape (but not the exact numbers involved) of the partial fraction decompositions of:

(a)  $\frac{2x-6}{x(x+1)(x+2)} =$

(b)  $\frac{x^6}{(x+2)^2(x^2+1)} =$

**Solution.**

(a)  $\frac{2x-6}{x(x+1)(x+2)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+2}$

(b) Since the numerator degree is 6 but the denominator degree is 4, we have to do long division first. This will result in  $Ax^2 + Bx + C$  plus a remainder. Overall:

$$\frac{x^6}{(x+2)^2(x^2+1)} = Ax^2 + Bx + C + \frac{D}{x+2} + \frac{E}{(x+2)^2} + \frac{Fx+G}{x^2+1}$$

**Problem 2. (6 points)** Evaluate the following indefinite integral:  $\int \frac{3x-4}{x(x+2)} dx$

**Solution.** Partial fractions tells us that  $\frac{3x-4}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$  for some numbers  $A, B$  that we still need to find:

- To find  $A$  and  $B$  we multiply both sides with  $x(x+2)$  to clear denominators:

$$3x - 4 = (x+2)A + xB$$

- Set  $x=0$  to get  $-4=2A$  so that  $A=-2$ .  
Set  $x=-2$  to get  $-10=-2B$  so that  $B=5$ .

We therefore have  $\int \frac{3x-4}{x(x+2)} dx = \int \left( \frac{-2}{x} + \frac{5}{x+2} \right) dx = -2\ln|x| + 5\ln|x+2| + C$ .