Example 38. Consider the triangle with vertices $P=(1,1,1)$, $Q=(2,1,3)$ and $R=(3,-1,1)$.

(a) Find the area of the triangle.

(b) Find a unit vector perpendicular to the plane PQR.

Solution. See previous lecture sketch!

Lines and planes

describing **lines**	describing **planes**
• through 2 points **or:** 1 point & 1 direction (the latter easily translates to parametrizations)	• through 3 points **or:** 1 point & 2 directions (the latter easily translates to parametrizations)
• by equations $5x+3y=2$ (in 2D) or two equations in 3D (intersecting two planes!)	• by equations $5x+3y+4z=2$ (in 3D)
• by a parametrization $\boldsymbol{r}(t)=\underbrace{\boldsymbol{r}_0}_{\text{point}}+\underbrace{t}_{\text{parameter}}\underbrace{\boldsymbol{v}}_{\text{direction}}$	• by a parametrization $\boldsymbol{r}(t)=\underbrace{\boldsymbol{r}_0}_{\text{point}}+\underbrace{s}_{\text{param 1}}\underbrace{\boldsymbol{v}}_{\text{direction 1}}+\underbrace{t}_{\text{param 2}}\underbrace{\boldsymbol{w}}_{\text{direction 2}}$

Example 39. We are familiar with $y=mx+b$ describing a line in 2D. There is a slight problem though because vertical lines (like $x=2$) cannot be written in this form. However, every line in 2D can be written as $ax+by=c$. For instance, $5x+3y=2$ (which is the same as $10x+6y=4$).

Example 40. Moving on to 3D, what is described by the equation $5x+3y+4z=2$?

Solution. This is a plane (not a line!) and one way to see why it should be something 2-dimensional (like a plane) is to argue as we did in Lectures 4 and 5: we are working in 3-dimensional space; by specifying 1 equation (here, $5x+3y+4z=2$) as constraint, the dimension is reduced to $3-1=2$.

Comment. We can also describe lines in 3D by such equations, but now we need 2 equations (in order to reduce the dimension from 3 to 1)!

Example 41. Find a parametrization for the line through $A=(1,1,1)$, $B=(2,1,3)$.

Solution. 1 point & 1 direction: we can pick the point $A=(1,1,1)$ (B works just as well) and $\overrightarrow{AB}=\langle 1,0,2\rangle$.
We then get the **parametrization** $P(t)=(1,1,1)+t\,\langle 1,0,2\rangle$, where $t\in(-\infty,\infty)$ is the parameter.
[For any t, $P(t)$ is a point on our line. For instance, $P(0)=(1,1,1)$ is A, while $P(1)=(2,1,3)$ is B. Slightly more interestingly, $P(1/2)=(3/2,1,2)$ is the mid point of A and B. Make sure you can see how we get all points of our line by varying the values of t.]

Example 42. Find a parametrization for the line segment from $A=(1,1,1)$ to $B=(2,1,3)$.

Solution. We can use the parametrization from the previous example: $P(t)=(1,1,1)+t\,\langle 1,0,2\rangle$. However, this time, we restrict t to the values $t\in[0,1]$. [Why?!]

Example 43. Find a parametrization for the line through $B=(2,1,3)$, $C=(3,-1,1)$.

Solution. Let's pick the point $B=(2,1,3)$ and the direction $\overrightarrow{BC}=\langle 1,-2,-2\rangle$.
We then get the parametrization $P(t)=(2,1,3)+t\,\langle 1,-2,-2\rangle$ with $t\in(-\infty,\infty)$.

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