Midterm #2

MATH 227 — Calculus III
Friday, Apr 1

Please print your name:

Problem 1. Consider the function $f(x,y) = xy\cos(x+y)$. Determine the following:

(a) $f_x =$

(b) $f_{xy} =$

(c) $\nabla f =$

(d) The linearization of $f(x,y)$ at the point $(1,-1)$.

(e) In which direction does $f(x,y)$ at $(1,-1)$ increase most rapidly?

Problem 2. Write down a chain rule for $\dfrac{\partial}{\partial\theta} f$ for $f(x,y)$ with $x = r\cos\theta$ and $y = r\sin\theta$.

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Problem 3. Consider the function $f(x,y,z) = 2 + x^2 - yz$.

(a) Find the derivative of $f(x,y,z)$ at $(1,-1,2)$ in the direction $\boldsymbol{v} = \boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$.

(b) Find an equation for the plane tangent to the surface $f(x,y,z) = 5$ at the point $(1,-1,2)$.

Problem 4. Find all local extreme values and saddle points of the function $f(x,y) = 3y^2 - 2y^3 - 3x^2 + 6xy$.

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Problem 5. Consider the integral $\int_0^2 \int_0^{x^2} (1+2xy)\mathrm{d}y\mathrm{d}x$.

(a) Evaluate the integral.

(b) Interchange the order of integration. Do not evaluate this second integral.

Problem 6. Convert the cartesian integral $\int_0^2 \int_0^{\sqrt{4-x^2}} \frac{1}{1+x^2+y^2}\,\mathrm{d}y\mathrm{d}x$ into an equivalent polar integral.

Do not evaluate either of these integrals.

Problem 7. Determine a system of equations for finding the extreme values of $f(x,y,z) = x - y + 2z$ on the sphere $x^2+y^2+z^2 = 3$. Do not attempt to solve this system of equations.

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