Midterm #2

MATH 238 — Differential Equations I
Wednesday, Nov 13, 2024

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 32 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (10 points) Determine the general solution of the following system:

$$\begin{aligned} y_1' &= 3y_1 + y_2 \\ y_2' &= y_1 + 3y_2 - 2e^x \end{aligned}$$

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Problem 2. (6 points) The mixtures in two tanks T_1, T_2 are kept uniform by stirring. Brine containing 3 lb of salt per gallon enters the first tank at a rate of 10 gal/min. Mixed solution from T_1 is pumped into T_2 at a rate of 8 gal/min, and also from T_2 into T_1 at a rate of 5 gal/min. Initially, tank T_1 is filled with 25 gal water containing 4 lb salt, and tank T_2 with 20 gal pure water.

Denote by $y_i(t)$ the amount (in pounds) of salt in tank T_i at time t (in minutes). Derive a system of linear differential equations for the y_i, including initial conditions. (Do *not* attempt to solve the system.)

Problem 3. (4 points) Assume that the angle $\theta(t)$ of a swinging pendulum is described by $\theta'' + 25\theta = 0$. Suppose $\theta(0) = 3$, $\theta'(0) = -5$. What are the period and the amplitude of the resulting oscillations?

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Problem 4. (4 points) Let L be a linear differential operator of order 4 with constant real coefficients. Suppose that $2-3i$ is a repeated characteristic root of L.

(a) What is the general solution to $Ly=0$?

(b) Write down the simplest form of a particular solution y_p of the DE $Ly=2xe^x-5e^{2x}\sin(3x)$ with undetermined coefficients.

Problem 5. (4 points) The position $y(t)$ of a certain mass on a spring is described by $3y''+dy'+5y=F\cos(\omega t)$.

(a) Assume first that there is no external force, i.e. $F=0$. For which values of d is the system underdamped?

(b) Now, $F\neq 0$ and the system is undamped, i.e. $d=0$. For which values of ω, if any, does resonance occur?

Problem 6. (4 points) Consider the following system of initial value problems:

$$\begin{array}{l} y_1''+3y_1=2y_2'+4 \\ y_2''+5y_2=6y_1' \end{array} \qquad y_1(0)=7,\ y_1'(0)=0,\ y_2(0)=6,\ y_2'(0)=-1$$

Write it as a first-order initial value problem in the form $\boldsymbol{y}'=M\boldsymbol{y}+\boldsymbol{f},\ \boldsymbol{y}(0)=\boldsymbol{c}$.

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(extra scratch paper)