# Preparing Midterm #1

**Problem 1.** Without solving the equation $(y-2)y' = x + e^y + 1$, answer the following questions.

  (a) Does the existence/uniqueness theorem guarantee the existence of a solution to the above equation with initial condition $y(2) = 0$? If so, does it guarantee the solution to be unique?

  (b) Same question for the initial condition $y(0) = 2$.

  (c) Sketch the slope field of the differential equation. What does it suggest regarding the previous questions?

  (d) Consider the solution with initial condition $y(1) = 0$. Find the equation for its tangent line at the point $(1, 0)$.

  (e) Again, considering the solution with initial condition $y(1) = 0$, what is $y''(1)$?

**Problem 2.** Solve the initial value problem $y' = 2xy + 3x^2 e^{x^2}$, $y(0) = 5$.

**Problem 3.** Find a general solution of the equation $x(x+y)y' = y(3x+y)$.

**Problem 4.** Find a general solution of the equation $2 + \dfrac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{2x+y}$.

**Problem 5.** In a city with a fixed population $P$, the time rate of change of the number $N$ of people who have heard a certain rumor is proportional to $N$ and $P - N$. Suppose initially 10% have heard the rumor and after a week this number has grown to 20%. What percentage will this number reach after one more week?

**Problem 6.** Solve the initial value problem $y'' - 5y' + 6y = 0$, $y(0) = 0, y'(0) = 1$.

**Problem 7.** Solve:   $x^2 \dfrac{\mathrm{d}y}{\mathrm{d}x} = xy - x^2 e^{y/x}$,   $y(1) = 0$

**Problem 8.** Find a general solution of the equation $xy' = y + x^2 \cos(x)$.

**Problem 9.** Find the general solution to $y^{(5)} - 4y^{(4)} + 5y''' - 2y'' = 0$.

**Problem 10.** Write down a homogeneous linear differential equation satisfied by $y(x) = 1 - 5x^2 e^{-2x}$.

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