

Please print your name:

Bonus challenge. Let me know about any typos you spot in the posted solutions (or lecture sketches). Any typo, that is not yet fixed by the time you send it to me, is worth a bonus point.

Problem 1. Find all eigenfunctions and eigenvalues of

$$y'' + \lambda y = 0, \quad y'(0) = 0, \quad y'(3) = 0.$$

Problem 2. Find the solution $u(x, t)$, for $0 < x < 3$ and $t \geq 0$, to the heat conduction problem

$$u_t = 5u_{xx}, \quad u_x(0, t) = u_x(3, t) = 0, \quad u(x, 0) = 7 + 4\cos(\pi x).$$

Derive your solution using separation of variables (at some step you may refer to the previous problem).

Problem 3. Find the solution $u(x, t)$, for $0 < x < 3$ and $t \geq 0$, to the heat conduction problem

$$2u_t = u_{xx}, \quad u_x(0, t) = 0, \quad u(3, t) = 0, \quad u(x, 0) = 2\cos\left(\frac{\pi x}{2}\right) + 7\cos\left(\frac{3\pi x}{2}\right).$$

Derive your solution using separation of variables (at some step you may refer to the EVP on the practice problems for the second midterm).

Problem 4. For $t \geq 0$ and $x \in [0, 4]$, consider the heat flow problem:

$$\begin{aligned} u_t &= 2u_{xx} + e^{-x/2} \\ u_x(0, t) &= 3 \\ u(4, t) &= -2 \\ u(x, 0) &= f(x) \end{aligned}$$

Determine the steady-state solution and spell out equations characterizing the transient solution.

Problem 5. Find the solution $u(x, y)$, for $0 < x < 3$ and $0 < y < 2$, to

$$u_{xx} + u_{yy} = 0, \quad u(x, 0) = 0, \quad u(x, 2) = 0, \quad u(0, y) = \sin(\pi y) + 2\sin(7\pi y), \quad u(3, y) = 0.$$

Derive your solution using separation of variables.