Midterm #1 – Practice

MATH 332 — Differential Equations II
Tuesday, Oct 1

Please print your name:

Problem 1.

(a) Find the general solution to $y^{(5)} - 4y^{(4)} + 5y''' - 2y'' = 0$.

(b) Find the general solution to $y''' - y = e^x + 7$.

(c) Solve $y'' + 2y' + y = 2e^{2x} + e^{-x}$, $y(0) = -1$, $y'(0) = 2$.

(d) Find the general solution to $y'' - 4y' + 4y = 3e^{2x}$.

(e) Consider a homogeneous linear differential equation with constant real coefficients which has order 6. Suppose $y(x) = x^2 e^{2x} \cos(x)$ is a solution. Write down the general solution.

(f) Write down a homogeneous linear differential equation satisfied by $y(x) = 1 - 5x^2 e^{-2x}$.

(g) Let y_p be any solution to the inhomogeneous linear differential equation $y'' + xy = e^x$. Find a homogeneous linear differential equation which y_p solves. *Hint:* Do not attempt to solve the DE.

Problem 2.

(a) Write down a (homogeneous linear) recurrence equation satisfied by $a_n = 3^n - 2^n$.

(b) Write down a (homogeneous linear) recurrence equation satisfied by $a_n = n^2 3^n - 2^n$.

Problem 3. Consider the sequence a_n defined by $a_{n+2} = a_{n+1} + 6a_n$ and $a_0 = 3$, $a_1 = -1$.

(a) Determine the first few terms of the sequence.

(b) Find a Binet-like formula for a_n.

(c) Determine $\lim_{n \to \infty} \frac{a_{n+1}}{a_n}$.

Problem 4. Let $M = \begin{bmatrix} 1 & 4 \\ 6 & -1 \end{bmatrix}$.

(a) Determine the general solution to $\boldsymbol{a}_{n+1} = M\boldsymbol{a}_n$.

(b) Determine a fundamental matrix solution to $\boldsymbol{a}_{n+1} = M\boldsymbol{a}_n$.

(c) Compute M^n.

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Problem 5.

(a) Write the differential equation $y''' + 7y'' - 3y' + y = 0$ as a system of (first-order) differential equations.

(b) Consider the following system of initial value problems:

$$\begin{aligned} y_1'' &= 3y_1' + 2y_2' - 5y_1 \\ y_2'' &= y_1' - y_2' + 3y_2 \end{aligned} \qquad y_1(0) = 1, \ y_1'(0) = -2, \ y_2(0) = 3, \ y_2'(0) = 0$$

Write it as a first-order initial value problem in the form $\boldsymbol{y}' = M\boldsymbol{y}$, $\boldsymbol{y}(0) = \boldsymbol{y}_0$.

Problem 6. Let $M = \begin{bmatrix} 11 & -2 \\ 3 & 4 \end{bmatrix}$.

(a) Determine the general solution to $\boldsymbol{y}' = M\boldsymbol{y}$.

(b) Determine a fundamental matrix solution to $\boldsymbol{y}' = M\boldsymbol{y}$.

(c) Compute e^{Mx}.

(d) Solve the initial value problem $\boldsymbol{y}' = M\boldsymbol{y}$ with $\boldsymbol{y}(0) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

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