

# Crash course in linear algebra, I

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

row vector  
column vector

2x3 matrix

vector operations:

- addition
- scaling

## matrix multiplication

$$\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = ax + by + cz$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 7 \\ -5 \end{bmatrix}$$

1·1 + (-1)·(-1) + 1·2  
1·0 + (-1)·(-1) + 1·(-2)

i<sup>th</sup> row times j<sup>th</sup> col → entry (i,j)  
row i, col j

$$\begin{aligned} 2x_1 + 7x_2 &= 3 \\ 3x_1 - x_2 &= 5 \end{aligned} \rightarrow \begin{bmatrix} 2 & 7 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

## identity matrix

$$AI = A = IA$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- associative law:  $A(BC) = (AB)C = ABC$  no ambiguity
- distributive law:  $A(B+C) = AB+AC$  and other way around
- commutative law **does not hold!**

$$\begin{bmatrix} 2 & 4 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 6 & 4 \end{bmatrix}$$

scales rows  
scales cols

## matrix transpose

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$