

# Crash course in linear algebra, II

EG

$$\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$2 \cdot 1 + 1 \cdot (-2)$

2x2 identity

$$AB = I \rightarrow B = A^{-1}$$

$A^{-1}$  = (matrix) inverse of  $A$

characterized by  $A^{-1}A = I = AA^{-1}$

$$\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\text{determinant } ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

provided if  $ad-bc \neq 0$

EG

$$\begin{aligned} 7x_1 - 2x_2 &= 3 \\ 2x_1 + x_2 &= 5 \end{aligned}$$

$$\Leftrightarrow \begin{bmatrix} 7 & -2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\begin{aligned} A^{-1} A \vec{x} &= \vec{b} \\ \vec{x} &= A^{-1} \vec{b} \end{aligned}$$

$$\vec{x} = \begin{bmatrix} 7 & -2 \\ 2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$= \frac{1}{11} \begin{bmatrix} 1 & 2 \\ -2 & 7 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 13 \\ 29 \end{bmatrix}$$

$7 \cdot 1 - (-2) \cdot 2$

$$x_1 = \frac{13}{11}$$

$$x_2 = \frac{29}{11}$$

$\det(A)$  = determinant of  $A$

or:  $|A|$

$\det(A) \neq 0$  ( $\Leftrightarrow$ )  $A$  is invertible

$\Leftrightarrow Ax = b$  has a (unique) solution for all  $b$

$$\vec{x} = A^{-1} \vec{b}$$

EG

$$\det \left( \begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = ad - bc$$