Review: DEs we can solve

EG $y' = 3y$ $\quad y(0) = 5$ IVP

initial condition

$y(x) = C e^{3x}$

general solution

unique solution:

$y(x) = 5e^{2x}$

EG $\overset{\frac{dy}{dx}}{y'} = x y^2$ non-linear DE

separable!

$$\frac{1}{y^2}\,dy = x\,dx \qquad \int y^a\,dy = \frac{1}{a+1}y^{a+1} + C$$

$$-\frac{1}{y} = \frac{1}{2}x^2 + C$$

$$y(x) = -\frac{1}{\frac{1}{2}x^2 + C} = \frac{2}{D - x^2}$$

$-2C$ $\quad D \to \infty$

general solution

missing solution: $y(x) = 0$ (singular solution)

EG $y' = x^2 y$ linear DE order 1

separable!

$$\frac{1}{y}\,dy = x^2\,dx \qquad [\,y(x) = 0 \text{ lost!}\,]$$

$$\ln|y| = \frac{1}{3}x^3 + C$$

$$|y| = e^{\frac{1}{3}x^3 + C}$$

never zero!

$D = 0$

$$y(x) = \underbrace{\pm e^C}_{D} e^{\frac{1}{3}x^3} = D e^{\frac{1}{3}x^3}$$

general solution

Euler's identity

$i^2 = -1$

$$e^{ix} = \cos(x) + i\sin(x)$$

$x = \pi$: $e^{\pi i} = -1$

or: $e^{\pi i} + 1 = 0$

Pf both sides are the (unique!) solution to the IVP $\quad y' = iy \quad y(0) = 1$