Homogeneous LDEs with constant coefficients

EG $y'' - y' - 2y = 0$ order 2

Solution ①: try $y(x) = e^{rx}$

plug into DE: $r^2 e^{rx} - r e^{rx} - 2e^{rx} = 0$ $\neq 0$

$r^2 - r - 2 = 0$

characteristic polynomial

$\rightsquigarrow r = 2, \quad r = -1$

$y_1(x) = e^{2x} \qquad y_2(x) = e^{-x}$

general solution: $c_1 e^{2x} + c_2 e^{-x}$

Solution ②: using $D = \frac{d}{dx}$

rewrite DE as: $\underbrace{(D^2 - D - 2)}_{\text{characteristic polynomial}} y = 0$

$(D-2)(D+1)\, y = 0$

get solutions from: $(D-2)y = 0$ $y' - 2y$

$\Rightarrow y_1(x) = e^{2x}$

$(D+1)y = 0$ $y' + y$

$\Rightarrow y_2(x) = e^{-x}$

EG $y'' - y' - 2y = 0 \qquad y(0) = 4 \qquad y'(0) = 5$

general solution: $C_1 e^{2x} + C_2 e^{-x} = y(x)$

$C_1 + C_2 = 4$

$2C_1 e^{2x} - C_2 e^{-x} = y'(x)$

$2C_1 - C_2 = 5$

$\Rightarrow C_1 = 3, \quad C_2 = 1$

unique solution to IVP:

$y(x) = 3e^{2x} + e^{-x}$