

Systems of differential equations

EG $y'' + 7y' - 5y = 0$

Write as system.

write $y_1 = y$, $y_2 = y'$

translates to $\begin{cases} y_1' = y_2 \\ y_2' = 5y_1 - 7y_2 \end{cases}$

in matrix form: $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ 5 & -7 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

$$y' = M y$$

EG $y_1'' = 2y_1' - 3y_2' + 7y_2$

$$y_1(0) = 2 \quad y_1'(0) = 3$$

$$y_2'' = 4y_1' + y_2' - 5y_1$$

$$y_2(0) = -1 \quad y_2'(0) = 1$$

write $y_3 = y_1'$, $y_4 = y_2'$

translates to: $\begin{cases} y_1' = y_3 \\ y_2' = y_4 \\ y_3' = 2y_3 - 3y_4 + 7y_2 \\ y_4' = 4y_3 + y_4 - 5y_1 \end{cases}$

matrix form: $\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}' = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 7 & 2 & -3 \\ -5 & 0 & 4 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$

$$y' = M y$$