Modeling

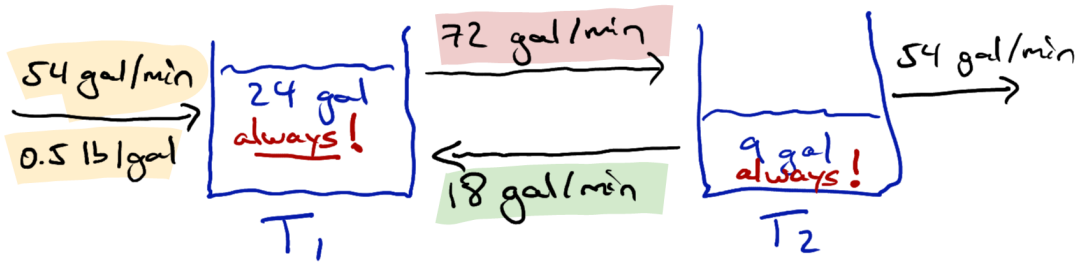
EG Two brine tanks:

T_1 : 24 gal water containing 3 lb salt

T_2 : 9 gal pure water

- T_1 being filled with 54 gal/min water containing 0.5 lb/gal salt
- 72 gal/min well-mixed solution flows out of T_1 into T_2
- 18 gal/min well-mixed solution flows out of T_2 into T_1
- 54 gal/min well-mixed solution is leaving T_2.

How much salt is in the tanks after t min?



$y_i(t)$ lb salt in tank T_i after t min

in time interval $[t, t+\Delta t]$:

$$\Delta y_1 \approx \underbrace{54 \cdot 0.5}_{\text{lb/min}} \cdot \Delta t - 72 \cdot \underbrace{\frac{y_1}{24}}_{\text{concentration lb/gal of salt in } T_1} \cdot \Delta t + 18 \cdot \frac{y_2}{9} \cdot \Delta t$$

$$y_1' = 27 - 3y_1 + 2y_2 \qquad y_1(0) = 3$$

$$\Delta y_2 \approx 72 \cdot \frac{y_1}{24} \cdot \Delta t - (18+54) \cdot \frac{y_2}{9} \cdot \Delta t$$

$$y_2' = 3y_1 - 8y_2 \qquad y_2(0) = 0$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}' = \begin{bmatrix} -3 & 2 \\ 3 & -8 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 27 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}(0) = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$y' = A\,y + f \qquad y(0) = c$$

HW: solve!