

# Power series + linear DEs

EG  $y'' = xy$      $y(0) = a, y'(0) = b$   
Airy equation

previously:  $y = a + bx + \frac{a}{6}x^3 + \frac{b}{12}x^4 + \dots$

now: power series solution

$$y = \sum_{n=0}^{\infty} a_n x^n \quad \text{with recursive formula for coefficients } a_n$$

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} (n-1)n a_n x^{n-2} = \sum_{n=0}^{\infty} (n+1)(n+2) a_{n+2} x^n$$

$$xy = \sum_{n=0}^{\infty} a_n x^{n+1} = \sum_{n=1}^{\infty} a_{n-1} x^n$$

compare coeffs of  $x^n$ :

for  $n \geq 1$ :  $(n+1)(n+2)a_{n+2} = a_{n-1}$

$n=0$ :  $1 \cdot 2 a_2 = 0$

$n \rightarrow n-2$   
 $(n-1)n a_n = a_{n-3}$  for  $n \geq 3$

$\Rightarrow a_0 = a, a_1 = b, a_2 = 0$   
 $a_n = \frac{1}{(n-1)n} a_{n-3}$  for  $n \geq 3$