Power series of familiar functions

EG $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

EG $\cosh(x) = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$

$\frac{1}{2}(e^x + e^{-x})$

even part of e^x

$\cosh(ix) = \cos(x)$

$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

EG $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$

geometric series

converges if $|x|<1$

diverges if $|x|>1$

radius of convergence $= 1$

$y = \frac{1}{1-x}$ singular point $x=1$

EG $\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-x^2)^n = \sum_{n=0}^{\infty} (-1)^n x^{2n}$

geometric series with $-x^2$ instead of x

EG $\arctan(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$

$= \int \frac{1}{1+x^2}\,dx + C = \int \sum_{n=0}^{\infty} (-1)^n x^{2n}\,dx + C$

$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} + \underbrace{C}_{=0}$

set $x=0$

$\arctan(0) = 0$

EG $\ln(x+1) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}$

HW $\ln(x+1) = \int \frac{1}{1+x}\,dx + C$

$\Leftrightarrow$ $\ln(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} (x-1)^{n+1}$

power series around $x=1$

$$\left[f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x-x_0)^n \right]$$

EG $\tan(x) = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \dots$

$y' = 1 + y^2 \quad y(0) = 0$ **HW**