PDEs: The heat equation

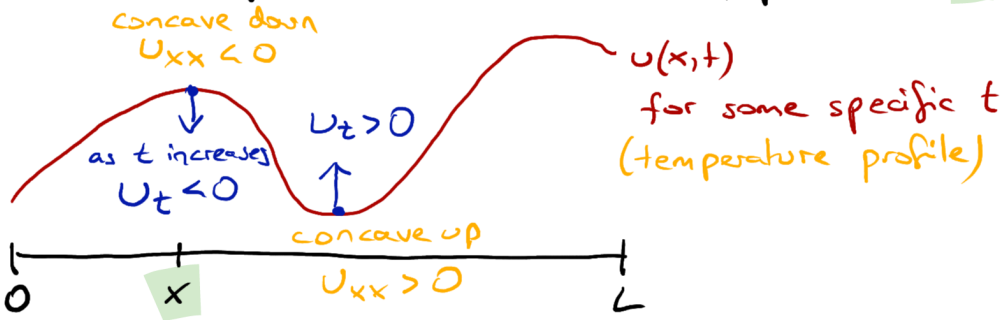
heat equation (one-dimensional)

$$u_t = k\, u_{xx} \qquad k > 0$$

$u = u(x,t)$
$u_t = \frac{\partial}{\partial t} u(x,t)$
$u_{xx} = \frac{\partial^2}{\partial x^2} u(x,t)$

This is a linear, homogeneous partial DE.
[=> if u_1, u_2 are solutions then so is $c_1 u_1 + c_2 u_2$]

model $u(x,t)$ = temperature at time t, position x



to describe a unique solution

- **initial condition** at $t = 0$:
  $u(x,0) = f(x)$ (initial temperature profile)
- **boundary condition** at $x = 0$ and $x = L$:
  EG
  - $u(0,t) = A$ $(= 0)$ $\quad u(L,t) = B$ $(= 0)$
  or:
  - $u_x(0,t) = 0 \quad u_x(L,t) = 0$

example solutions to $u_t = k\, u_{xx}$

- $u(x,t) = ax + b$ $\quad u_t = 0,\ u_{xx} = 0$
- $u(x,t) = e^{kt} e^{x}$ $\quad u_t = k u,\ u_x = u,\ u_{xx} = u$
- $u(x,t) = e^{-kt} \cos(x)$ $\quad u_t = -k u,\ u_{xx} = -u$ $\quad$ or: $e^{-kt} \sin(x)$
- $u(x,t) = e^{-k\lambda^2 t} \cos(\lambda x)$ $\quad u_t = -k\lambda^2 u,\ u_{xx} = -\lambda^2 u$