The Laplace equation

heat equation

$u_t = k \Delta u$

Δu: Laplacian of u

dim = 1 $\quad u_t = k u_{xx}$

dim = 2 $\quad u_t = k(u_{xx} + u_{yy})$

$u(t, x, y)$

dim = 3 $\quad u_t = k(u_{xx} + u_{yy} + u_{zz})$

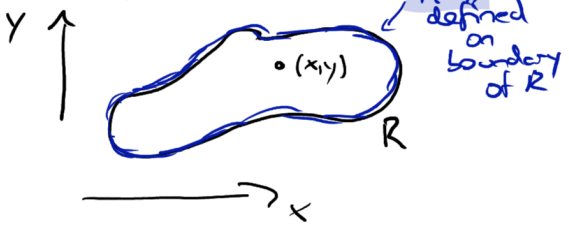
steady-state temperature

$u_t = 0$

$\Delta u = 0$

Laplace equation

dim = 2



$u_{xx} + u_{yy} = 0$ within region R — PDE

$u(x,y) = f(x,y)$ on boundary of R — BC

Dirichlet problem

EG

$u_{xx} + u_{yy} = 0$ — PDE

for $0<x<a$ and $0<y<b$

$u(x,0) = f_1(x)$

$u(0,y) = f_2(y)$

$u(x,b) = f_3(x)$

$u(a,y) = f_4(y)$

BC

split up into 4 Dirichlet problems like:

$u_{xx} + u_{yy} = 0$ homogeneous

$u(x,0) = 0$

$u(0,y) = 0$

$u(x,b) = f_3(x)$

$u(a,y) = 0$

Solve using separation of variables

$u(x,y) = X(x)\,Y(y)$

sum of 4 solutions ⟶ overall solution