Midterm #2

THU, NOV 5

exam: 2^{00} – 3^{15} PM

upload work by 3^{45} PM

PDF

format

- as last time
- show-your-work problems ~3
- short answer problems ~6

practice

- review HW
- practice problems + solutions

tools

- calculators allowed but: show work
- notes allowed but: watch time

Questions?

- systems of DEs
  - matrix exponential
  - inhomogeneous systems
  - modeling

- power series
  - solutions of nonlinear DEs
  - solutions of linear DEs (recursive!)
  - minimal radius of convergence
  - of familiar functions
  - of rational functions (recursive!)

- Fourier series
  - resonance
  - inhomogeneous linear DEs with constant coeffs
  - Fourier cosine + sine series

- Boundary value problems
  - eigenvalue problems

[next: used in PDEs]

$$3y'' + 48y = \sum_{n=1}^{\infty} \frac{1}{n^2+8} \cos(nt)$$

$3D^2 + 48 = 0$

$D^2 + 16 = 0$ "old" roots : $\pm 4i$

$$3y'' + 48y = \cos(nt)$$

"new" roots : $\pm in$

resonance if $n = 4$

$n \neq 4$: $y_p = A\cos(nt) + B\sin(nt)$

$n = 4$: $y_p = At\cos(4t) + Bt\sin(4t)$

$\lambda = 4$

$y_p =$ [ ] $+ \sum_{\substack{n=1 \\ n \neq \lambda}}^{\infty}$ ---

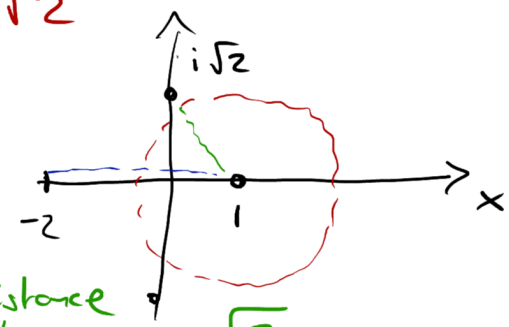
EG radius of convergence R

$$(x^2+2)y''' - \frac{7x}{x+2}y' + 8y = 0$$

$y(1) = \ldots$ $y'(1) = \ldots$ $y''(1) = \ldots$

singular points : $-2, \pm i\sqrt{2}$

$y''' = \frac{1}{x^2+2}[\ldots]$



$R \geq$ minimum distance from 1 to a singular point $= \sqrt{3}$

distance 1 to $\pm i\sqrt{2} = |1 - i\sqrt{2}| = \sqrt{1^2 + (\sqrt{2})^2} = \sqrt{3}$

1 to $-2 = 3$

EG

$$y(x) = \frac{2-3x}{1+4x-7x^2} = \sum_{n=0}^{\infty} a_n x^n$$

$$2-3x = (1+4x-7x^2)\sum_{n=0}^{\infty} a_n x^n$$

$$= \sum_{n=0}^{\infty} a_n x^n + 4\sum_{n=0}^{\infty} a_n x^{n+1} - 7\sum_{n=0}^{\infty} a_n x^{n+2}$$

$$= \sum_{n=0}^{\infty} a_n x^n + 4\sum_{n=1}^{\infty} a_{n-1} x^n - 7\sum_{n=2}^{\infty} a_{n-2} x^n$$

compare coeff's of x^n:

$n=0$: $2 = a_0$ $\rightsquigarrow$ $a_0 = 2$

$n=1$: $-3 = a_1 + 4a_0$ $\rightsquigarrow$ $a_1 = -3-4a_0 = -11$

$n \geq 2$: $0 = a_n + 4a_{n-1} - 7a_{n-2}$

$$a_n = -4a_{n-1} + 7a_{n-2}$$