

Preparation problems for the discussion sections on November 18th and 20th

1. For each of the following matrices, determine the eigenvalues of the matrix and for each eigenvalue, determine (a basis for) the eigenspace that is associated to that eigenvalue.

(a) $\begin{bmatrix} 4 & 0 & -2 \\ 1 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix},$

(b) $\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix},$

(c) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$

2. Let

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 3 \end{bmatrix}.$$

Determine the eigenvalues of A, B, C and, for each eigenvalue, determine the eigenspace that is associated to that eigenvalue.

3. (This question is not yet relevant for the third midterm exam.) Let

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

Find a diagonal matrix D and an invertible matrix P such that $A = PDP^{-1}$.