Math 415 - Midterm 3

Thursday, November 20, 2014

Circle your section:

Philipp Hieronymi	2pm	3pm
Armin Straub	9am	11am

Name:

NetID:

UIN:

Problem 0. ***[1 point]*** Write down the number of your discussion section (for instance, AD2 or ADH) and the first name of your TA (Allen, Anton, Mahmood, Michael, Nathan, Pouyan, Tigran, Travis).

Section:	TA:

To be completed by the grader:

0	1	2	3	4	5	MC	$\sum$
/1	/7	/8	/8	/8	/8	/20	/60

Good luck!

Instructions

- No notes, personal aids or calculators are permitted.
- This exam consists of 10 pages. Take a moment to make sure you have all pages.
- You have 75 minutes.
- Answer all questions in the space provided. If you require more space to write your answer, you may continue on the back of the page (make it clear if you do).
- **Explain your work!** Little or no points will be given for a correct answer with no explanation of how you got it.
- In particular, you have to **write down all row operations** for full credit.

Problem 1. ***[7 points]*** Find the QR decomposition of $A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$.

Solution. We start with the columns of $A(= [\mathbf{v}_1\mathbf{v}_2])$ and we use Gram-Schmidt to find the columns of $Q(= [\mathbf{q}_1\mathbf{q}_2])$:

$$q_1 = \frac{\mathbf{v}_1}{\|\mathbf{v}_1\|} = \frac{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}{\|\begin{bmatrix} 1 \\ 1 \end{bmatrix}\|} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

and

$$\mathbf{q}_2 = \frac{\mathbf{v}_2 - (\mathbf{q}_1 \cdot \mathbf{v}_2)\mathbf{q}_1}{\|\mathbf{v}_2 - (\mathbf{q}_1 \cdot \mathbf{v}_2)\mathbf{q}_1\|} = \frac{\begin{bmatrix} 2 \\ 0 \end{bmatrix} - (\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 0 \end{bmatrix}) \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}}{\|\begin{bmatrix} 2 \\ 0 \end{bmatrix} - (\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 0 \end{bmatrix}) \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}\|} = \frac{\begin{bmatrix} 1 \\ -1 \end{bmatrix}}{\|\begin{bmatrix} 1 \\ -1 \end{bmatrix}\|} = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}.$$

Hence,

$$Q = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

Finally:

$$R = Q^T A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{2} \end{bmatrix}$$

Problem 2. ***[8 points]*** Let $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Find the eigenvalues of A, as well as a basis for the corresponding eigenspaces.

Solution. If we expand along the last row, we obtain:

$$\det(\begin{bmatrix} 2-\lambda & 0 & 1 \\ 2 & 1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix}) == (1-\lambda)\det(\begin{bmatrix} 2-\lambda & 0 \\ 2 & 1-\lambda \end{bmatrix}) = (1-\lambda)^2(2-\lambda)$$

Hence, the eigenvalues of A are 1 (with multiplicity 2) and 2. For $\lambda = 1$:

$$\begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R2 \to R2 - 2R1} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R1 \to R1 + 1/2R1, R2 \to -1/2R2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Since there is only one free variable (there could have been up to 2 because the eigenvalue has multiplicity 2), the corresponding eigenspace is just one-dimensional and has basis $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

For $\lambda = 2$:

$$\begin{bmatrix} 0 & 0 & 1 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \xrightarrow{R2 \to 1/2R2, R3 \to R3+R1, R1 \leftrightarrow R2} \begin{bmatrix} 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence, the corresponding eigenspace has the basis $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$.

Problem 3. Let $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$.

(a) ***[6 points]*** Find the least squares solution of $A\boldsymbol{x} = \boldsymbol{b}$.
(b) ***[2 points]*** Find the least squares line for the data points $(-1, 1)$, $(0, -1)$, $(1, 1)$.

Solution. (a) We have to solve $A^T A\hat{\boldsymbol{x}} = A^T\boldsymbol{b}$:

$$A^T A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

and

$$A^T\boldsymbol{b} = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Since

$$\left[\begin{array}{cc|c} 3 & 0 & 1 \\ 0 & 2 & 0 \end{array}\right] \xrightarrow{R1 \to 1/3R1, R2 \to 1/2R2} \left[\begin{array}{cc|c} 1 & 0 & \frac{1}{3} \\ 0 & 1 & 0 \end{array}\right],$$

we obtain

$$\hat{\boldsymbol{x}} = \begin{bmatrix} \frac{1}{3} \\ 0 \end{bmatrix}.$$

(b) Writing the least squares line as $y = a + bx$, we have to find a and b so that $\begin{bmatrix} a \\ b \end{bmatrix}$ is the least squares solution of: (by changing the order of the data points)

$$A\boldsymbol{x} = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = \boldsymbol{b}$$

By part (a), we have:

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ 0 \end{bmatrix}.$$

Hence, the least squares line is $y = \frac{1}{3}$.

Problem 4. Find the determinant of the following matrices. Show all steps of your calculations.

(a) ***[4 points]***

$$\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

(b) ***[4 points]***

$$\begin{bmatrix} 2 & -1 & 3 & 7 \\ 0 & 1 & 0 & 1 \\ 2 & -1 & 3 & 2 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

[*Hint:* For this second matrix, begin your calculation with a row operation to save time.]

Solution. (a) We expand the determinant along the third column:

$$\det\left(\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 1 & 1 & 0 & 0 \end{bmatrix}\right) = 3\det\left(\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix}\right) = 3\det\left(\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}\right) = 3.2 = 6$$

(b) We use row operations to transform the matrix, A, into an upper triangular matrix, B:

$$A = \begin{bmatrix} 2 & -1 & 3 & 7 \\ 0 & 1 & 0 & 1 \\ 2 & -1 & 3 & 2 \\ 0 & 1 & 1 & 0 \end{bmatrix} \xrightarrow{R3\to R3-R1, R4\to R4-R2, R3\leftrightarrow R4} \begin{bmatrix} 2 & -1 & 3 & 7 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -5 \end{bmatrix} = B$$

Since we swap rows once, we have $\det(A) = -\det(B)$. Hence,

$$\det(A) = -\det(B) = -(2.1.1.(-5)) = 10.$$

Problem 5. Let $W = \operatorname{span}\left\{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}\right\}$.

(a) ***[6 points]*** Find the projection matrix corresponding to orthogonal projection onto W.

(b) ***[2 points]*** What is the orthogonal projection of $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ onto W?

Solution. (a) We have to find the orthogonal projection of elements of the standard basis onto W. Since $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ are orthogonal to each other, the orthogonal projection of the first standard basis vector onto W is

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}_W = \frac{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \frac{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}}{\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} \\ 0 \\ \frac{2}{5} \end{bmatrix}$$

and

$$\begin{bmatrix}0\\1\\0\end{bmatrix}_W = \frac{\begin{bmatrix}0\\1\\0\end{bmatrix}\cdot\begin{bmatrix}0\\1\\0\end{bmatrix}}{\begin{bmatrix}0\\1\\0\end{bmatrix}\cdot\begin{bmatrix}0\\1\\0\end{bmatrix}}\begin{bmatrix}0\\1\\0\end{bmatrix} + \frac{\begin{bmatrix}0\\1\\0\end{bmatrix}\cdot\begin{bmatrix}1\\0\\2\end{bmatrix}}{\begin{bmatrix}1\\0\\2\end{bmatrix}\cdot\begin{bmatrix}1\\0\\2\end{bmatrix}}\begin{bmatrix}1\\0\\2\end{bmatrix} = \begin{bmatrix}0\\1\\0\end{bmatrix}$$

and

$$\begin{bmatrix}0\\0\\1\end{bmatrix}_W = \frac{\begin{bmatrix}0\\0\\1\end{bmatrix}\cdot\begin{bmatrix}0\\1\\0\end{bmatrix}}{\begin{bmatrix}0\\1\\0\end{bmatrix}\cdot\begin{bmatrix}0\\1\\0\end{bmatrix}}\begin{bmatrix}0\\1\\0\end{bmatrix} + \frac{\begin{bmatrix}0\\0\\1\end{bmatrix}\cdot\begin{bmatrix}1\\0\\2\end{bmatrix}}{\begin{bmatrix}1\\0\\2\end{bmatrix}\cdot\begin{bmatrix}1\\0\\2\end{bmatrix}}\begin{bmatrix}1\\0\\2\end{bmatrix} = \begin{bmatrix}\frac{2}{5}\\0\\\frac{4}{5}\end{bmatrix}.$$

Hence, the projection matrix is:

$$A = \begin{bmatrix}\frac{1}{5} & 0 & \frac{2}{5}\\0 & 1 & 0\\\frac{2}{5} & 0 & \frac{4}{5}\end{bmatrix}$$

(b) The orthogonal projection is

$$\begin{bmatrix}1\\1\\0\end{bmatrix}_W = \begin{bmatrix}\frac{1}{5} & 0 & \frac{2}{5}\\0 & 1 & 0\\\frac{2}{5} & 0 & \frac{4}{5}\end{bmatrix}\begin{bmatrix}1\\1\\0\end{bmatrix} = \begin{bmatrix}\frac{1}{5}\\1\\\frac{2}{5}\end{bmatrix}.$$

MULTIPLE CHOICE

(10 questions, 2 points each)

Instructions for multiple choice questions

- No reason needs to be given. There is always exactly one correct answer.
- Enter your answer on the scantron sheet that is included with your exam. In addition, on your exam paper, circle the choices you made on the scantron sheet.
- Use a **number 2 pencil** to shade the bubbles completely and darkly.
- Do **NOT** cross out your mistakes, but rather erase them thoroughly before entering another answer.
- Before beginning, please code in your name, UIN, and netid in the appropriate places. In the **'Section' field on the scantron**, please enter

  000 if Armin Straub is your instructor,

  001 if Philipp Hieronymi is your instructor.

MC 1. Let $W = \text{span}\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$, and let $\boldsymbol{y} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}$.

Suppose that $\boldsymbol{y} = \boldsymbol{a} + \boldsymbol{b}$, where $\boldsymbol{a}$ is in W and $\boldsymbol{b}$ is orthogonal to W. Then:

(a) $\boldsymbol{a} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$

(b) $\boldsymbol{a} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$

(c) $\boldsymbol{a} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ and $\boldsymbol{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$

(d) none of the above

Solution. (a)

MC 2. Let Q be an orthogonal matrix. What is the best you can say about $\det(Q)$?

(a) $\det Q = 1$

(b) $\det Q = \pm 1$

(c) $\det Q \neq 0$

(d) $\det Q > 0$

Solution. (b), we have $Q^T Q = I$ so $\det(Q^T)\det(Q) = \det(Q)^2 = \det(I) = 1$. Hence, $\det(Q) = 1$ or -1 but it is not necessarily equal to 1 or necessarily equal to -1.

MC 3. If A and B are 2×2 matrices with $\det(A) = 2$ and $\det(B) = -1$. What is the determinant of $C = -2ABA^T$?

(a) 4
(b) -8
(c) 8
(d) -16
(e) 16

Solution. (d), we have:

$$\det(C) = \det(-2ABA^T) = (-2)^2 \det(A)\det(B)\det(A^T) = 4\det(A)^2\det(B) = -16$$

MC 4. Let A, B be two $n \times n$-matrices. Consider the following two statements:

(S1) If $\det(A) = 0$, then two rows or two columns of A are the same, or a row or a column of A is zero.
(S2) If two row interchanges on A are made in succession to get a matrix B, then $\det(A) = \det(B)$.

Then:

(a) Statement S1 and Statement S2 are correct.
(b) Only Statement S1 is correct.
(c) Only Statement S2 is correct.
(d) Neither Statement S1 nor Statement S2 is correct.

Solution. (c), consider $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix}$ to show that (S1) is not correct.

MC 5. Which of the following choices for a makes $\begin{bmatrix} 3 & a & 0 \\ 3a & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ invertible?

(a) any real number except $-\sqrt{2}$ and $\sqrt{2}$
(b) any real number except -2 and 2
(c) 6
(d) any real number except -2 and 6

Solution. (a), the matrix is invertible if and only if the determinant of the matrix is not zero. We have:

$$\det\left(\begin{bmatrix} 3 & a & 0 \\ 3a & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) = 6 - 3a^2 = 0 \Leftrightarrow a = \pm\sqrt{2}$$

Hence, the matrix is invertible for any real number except $-\sqrt{2}$ and $\sqrt{2}$.

MC 6. Consider the following two statements:

(T1) If $\{\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}\}$ is an orthogonal basis for W, then multiplying $\boldsymbol{v_3}$ by a scalar c gives a new orthogonal basis $\{\boldsymbol{v_1}, \boldsymbol{v_2}, c\boldsymbol{v_3}\}$.

(T2) The Gram–Schmidt process produces from a linearly independent set $\{\boldsymbol{v_1}, \ldots, \boldsymbol{v_n}\}$ an orthonormal set $\{\boldsymbol{q_1}, \ldots, \boldsymbol{q_n}\}$ with the property that for each $k \leq n$ the vectors $\{\boldsymbol{q_1}, \ldots, \boldsymbol{q_k}\}$ span the same subspace as $\{\boldsymbol{v_1}, \ldots, \boldsymbol{v_k}\}$.

Then:

(a) Statement T1 and Statement T2 are correct.
(b) Only Statement T1 is correct.
(c) Only Statement T2 is correct.
(d) Neither Statement T1 nor Statement T2 is correct.

Solution. Both (a) and (c) were counted as correct.
Technically, (c) is correct because (T1) is not always true (multiplying with $c = 0$ does not result in an orthogonal basis). On the other hand, (T1) would be a true statement if the scalar c is assumed to be nonzero.

MC 7. Consider the vector space V of all continuous functions $\mathbb{R} \to \mathbb{R}$, which are periodic with period 8. What is a natural inner product on V?

(a) $\langle f, g\rangle = \int_0^{2\pi} f(t)g(t)\mathrm{d}t$
(b) $\langle f, g\rangle = \int_0^{8} f(t)g(t)\mathrm{d}t$
(c) $\langle f, g\rangle = \int_0^{8} (f(t) - g(t))^2\mathrm{d}t$
(d) $\langle f, g\rangle = 8f(t)g(t)$
(e) $\langle f, g\rangle = f(1)g(1) + \ldots + f(n)g(n)$

Solution. (b)

MC 8. Consider the space $\mathbb{P}^3$ of polynomials of degree up to 3, together with the inner product

$$\langle p(t), q(t)\rangle = \int_0^1 p(t)q(t)\mathrm{d}t.$$

What is the orthogonal projection of the polynomial t onto $\operatorname{span}\{t^2\}$?

(a) $\dfrac{\int_0^1 s^3\mathrm{d}s}{\int_0^1 s^4\mathrm{d}s}\, t^2$
(b) $\dfrac{\int_0^1 s^3\mathrm{d}s}{\int_0^1 s^4\mathrm{d}s}\, t$
(c) 0
(d) t
(e) t^2

Solution. (a), let $W = \operatorname{span}\{t^2\}$. The orthogonal projection of t onto W is

$$t_W = \frac{\langle t, t^2\rangle}{\langle t^2, t^2\rangle} t^2 = \frac{\int_0^1 s^3\mathrm{d}s}{\int_0^1 s^4\mathrm{d}s}\, t^2.$$

MC 9. Let A be an $n \times n$ matrix. Consider the following two statements:

(U1) The matrix $7A$ has the same eigenvectors as A.
(U2) The matrix $7A$ has the same eigenvalues as A.

Then:

(a) Statement U1 and Statement U2 are correct.
(b) Only Statement U1 is correct.
(c) Only Statement U2 is correct.
(d) Neither Statement U1 nor Statement U2 is correct.

Solution. (b), if $\mathbf{v}$ is an eigenvector of A and $A\mathbf{v} = \lambda\mathbf{v}$, then $(7A)\mathbf{v} = (7\lambda)\mathbf{v}$. Hence, (U1) is correct.
Consider $A = I$ (identity matrix) to see that $U2$ is not correct. (The eigenvalues of $7A$ are 7 times the eigenvalues of A.)

MC 10. Let $W = \text{span}\left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$ and $\boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\boldsymbol{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$. Let $\boldsymbol{w}_1$ be the orthogonal projection of $\boldsymbol{v}_1$ onto W, and let $\boldsymbol{w}_2$ be the orthogonal projection of $\boldsymbol{v}_2$ onto W. Then:

(a) $\boldsymbol{w}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\boldsymbol{w}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

(b) $\boldsymbol{w}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\boldsymbol{w}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

(c) $\boldsymbol{w}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\boldsymbol{w}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(d) $\boldsymbol{w}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\boldsymbol{w}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(e) $\boldsymbol{w}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\boldsymbol{w}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

Solution. (c), $\boldsymbol{v}_1$ is in W and $\boldsymbol{v}_2$ is orthogonal to W hence $\boldsymbol{w}_1 = \boldsymbol{v}_1$ and $\boldsymbol{w}_2 = \mathbf{0}$.