

# Quiz #4

Please print your name:

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**Problem 1.** Let  $A = \begin{bmatrix} 1 & 2 & 0 & 2 & 2 \\ 1 & 2 & 1 & 0 & 0 \\ 2 & 4 & 0 & 4 & 4 \\ 1 & 2 & 1 & 0 & 0 \end{bmatrix}$ . Find a basis for each of  $\text{col}(A)$ ,  $\text{row}(A)$  and  $\text{null}(A)$ .

**Solution.** We eliminate:

$$\begin{bmatrix} 1 & 2 & 0 & 2 & 2 \\ 1 & 2 & 1 & 0 & 0 \\ 2 & 4 & 0 & 4 & 4 \\ 1 & 2 & 1 & 0 & 0 \end{bmatrix} \begin{array}{l} R_2 - R_1 \Rightarrow R_2 \\ R_3 - 2R_1 \Rightarrow R_3 \\ R_4 - R_1 \Rightarrow R_4 \\ \rightsquigarrow \end{array} \begin{bmatrix} 1 & 2 & 0 & 2 & 2 \\ 0 & 0 & 1 & -2 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & -2 \end{bmatrix} \begin{array}{l} \\ \\ R_4 - R_2 \Rightarrow R_4 \\ \rightsquigarrow \end{array} \begin{bmatrix} 1 & 2 & 0 & 2 & 2 \\ 0 & 0 & 1 & -2 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The general solution to  $A\mathbf{x} = \mathbf{0}$  is  $\mathbf{x} = \begin{bmatrix} -2s_1 - 2s_2 - 2s_3 \\ s_1 \\ 2s_2 + 2s_3 \\ s_2 \\ s_3 \end{bmatrix}$ .

• basis for  $\text{col}(A)$ :  $\begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$

• basis for  $\text{row}(A)$ :  $\begin{bmatrix} 1 \\ 2 \\ 0 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -2 \\ -2 \end{bmatrix}$

• basis for  $\text{null}(A)$ :  $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$

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