

Quiz #5

Please print your name:

Problem 1. Find the eigenvalues of $A = \begin{bmatrix} 3 & 4 & 1 \\ 0 & 2 & 0 \\ 1 & 4 & 3 \end{bmatrix}$ as well as bases for the eigenspaces.

Solution. By expanding by the second row, we find that the characteristic polynomial is

$$\begin{vmatrix} 3-\lambda & 4 & 1 \\ 0 & 2-\lambda & 0 \\ 1 & 4 & 3-\lambda \end{vmatrix} = (2-\lambda) \begin{vmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix} = (2-\lambda)[(3-\lambda)^2 - 1] = (2-\lambda)(\lambda-2)(\lambda-4).$$

Hence, the eigenvalues are $\lambda=2$ (with multiplicity 2) and $\lambda=4$.

- For $\lambda=4$, the eigenspace $\text{null}\left(\begin{bmatrix} -1 & 4 & 1 \\ 0 & -2 & 0 \\ 1 & 4 & -1 \end{bmatrix}\right)$ has basis $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.
- For $\lambda=2$, the eigenspace $\text{null}\left(\begin{bmatrix} 1 & 4 & 1 \\ 0 & 0 & 0 \\ 1 & 4 & 1 \end{bmatrix}\right)$ has basis $\begin{bmatrix} -4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$.

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