Euclidean algorithm

$$\gcd(30, 108) = 2 \cdot 3 = 6$$

greatest
common
divisor

$2 \cdot 3 \cdot 5$ $2^2 \cdot 3^3$

factorizations

BAD! (for large numbers)

RK   1991 : $100,000

135066 ... 300 digits ... 563

$18 = 108 - 3 \cdot 30$

$$\begin{aligned}
\gcd(30, 108) \\
= \gcd(18, 30) \\
= \gcd(12, 18) \\
= \gcd(6, 12) \\
= 6
\end{aligned}$$

$$\begin{aligned}
108 &= 3 \cdot 30 + 18 \\
30 &= 1 \cdot 18 + 12 \\
18 &= 1 \cdot 12 + 6 \\
12 &= 2 \cdot 6 + 0
\end{aligned}$$

Optimization : allow negative remainders

$$\begin{aligned}
\gcd(30, 108) \\
= \gcd(12, 30) \\
= \gcd(6, 12) \\
= 6
\end{aligned}$$

$$\begin{aligned}
108 &= 4 \cdot 30 - 12 \\
30 &= 2 \cdot 12 + 6 \\
12 &= 2 \cdot 6 + 0
\end{aligned}$$