Sets of residues, linear congruences

EG mod 5

complete sets of residues:

- $0, 1, 2, 3, 4$
- $0, \pm 1, \pm 2$

natural "reduced"

- $-5, 2, 9, -2, 6$
  $\quad 0 \quad 2 \quad 4 \quad 3 \quad 1$
- $0, 3, 3^2, 3^3, 3^4$
  $\quad 0 \quad 3 \quad 4 \quad 2 \quad 1$

"3 multiplicative generator mod 5"

EG mod 10

invertible residues: $1, 3, 7, 9$

x^{-1} exists mod 10 $\iff \gcd(x, 10) = 1$

[better: $\pm 1, \pm 3$]

EG How many solutions?

- $3x \equiv 2 \pmod 7$ $\qquad \gcd(3,7) = 1$

  1 solution: $x \equiv \underbrace{3^{-1}}_{5} \cdot 2 \pmod 7$, $\equiv 3$

- $3x \equiv 2 \pmod 9$ $\qquad \gcd(3,9) = 3$

  0 solutions

  $3x + 9y = 2$

  $x + 3y = \frac{2}{3}$ no integer solutions

- $6x \equiv 3 \pmod 9$ $\qquad \gcd(6,9) = 3$

  3 solutions: $x \equiv 2, 5, 8 \pmod 9$

  $6x + 9y = 3$

  $\iff 2x + 3y = 1$

  $\iff 2x \equiv 1 \pmod 3$

  $\iff x \equiv 2^{-1} \cdot 1 \pmod 3$, $\equiv 2$

  $\iff x \equiv 2, 5, 8 \pmod 9$

  $2, 5, 8, 11, 14, \ldots$

  $\pmod 9$ $\boxed{2, 5, 8}, 2, 5, \ldots$