Midterm #1: practice

MATH 311 — Intro to Number Theory
Midterm: Thursday, Oct 1

Please print your name:

Problem 1. Using the Euclidean algorithm, find the general solution to the diophantine equation $156x + 247y = 65$.

(Use Homework Problems 1.4, 1.5, 1.6, 2.2 to generate more practice problems of this kind.)

Problem 2.

(a) For which values of k has the diophantine equation $123x + 360y = k$ at least one integer solution?

(b) Determine the general solution to the diophantine equation $123x + 360y = 99$.

(c) Determine all solutions to $123x + 360y = 99$ with x and y positive integers.

(Use Homework Problems 2.1, 2.2, 2.3 to generate more practice problems of this kind.)

Problem 3.

(a) Determine $31^{4441} \pmod{23}$, carefully showing all steps.

(b) Is $314^{159} + 265^{358} + 10$ divisible by 19?

(Use Homework Problems 3.8, 3.9 to generate more practice problems of this kind.)

Problem 4.

(a) Find the modular inverse of 17 modulo 23.

(b) Solve $15x \equiv 7 \pmod{31}$.

(c) List all invertible residues modulo 10.

(d) How many solutions does $16x \equiv 1 \pmod{70}$ have modulo 70? Find all solutions.

(e) How many solutions does $16x \equiv 4 \pmod{70}$ have modulo 70? Find all solutions.

(Use Homework Problems 2.8, 2.9, 2.10, 3.1, 3.2 to generate more practice problems of this kind.)

Problem 5. Solve the following system of congruences:

$$\begin{aligned} 3x + 5y &\equiv 6 \pmod{25} \\ 2x + 7y &\equiv 2 \pmod{25} \end{aligned}$$

(Use Homework Problems 3.4, 3.5 to generate more practice problems of this kind.)

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Problem 6. Spell out a precise version of the following famous results:

(a) Bézout's identity

(b) Prime number theorem

(c) Fermat's little theorem

Problem 7.

(a) Let a, n be positive integers. Show that a has a modular inverse modulo n if and only if $\gcd(a, n) = 1$.

(b) Let p be a prime, and a an integer such that $p \nmid a$. Show that the modular inverse a^{-1} exists, and that

$$a^{-1} \equiv a^{p-2} \pmod{p}.$$

(c) Compute $17^{-1} \pmod{101}$ in two different ways:

- Using the Euclidean algorithm.
- Using the previous part of this problem and binary exponentiation.

Problem 8.

(a) Determine $\operatorname{lcm}(81, 135)$.

(Use Homework Problem 1.7 to generate more practice problems of this kind.)

(b) The residues $-2, -9, 6, 17, -10$ do not form a complete set of residues modulo 6. Which residue is missing?

(Use Homework Problem 3.3 to generate more practice problems of this kind.)

(c) Express 3141 in base 6.

(Use Homework Problems 3.6, 3.7 to generate more practice problems of this kind.)

(d) Determine, without the help of a calculator, the remainder of 112358132134 modulo 9.

(Use Homework Problem 3.10 to generate more practice problems of this kind.)

(e) What is the remainder of 62831853 modulo 11?

(Use Homework Problem 3.11 to generate more practice problems of this kind.)

Problem 9.

(a) Solve $x \equiv 2 \pmod{11}$, $x \equiv 3 \pmod{13}$.

(b) Using the Chinese remainder theorem, determine all solutions to $x^2 \equiv 4 \pmod{55}$.

(Use Homework Problems 4.1, 4.2, 4.3, 4.4 to generate more practice problems of this kind.)

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